APPLICATION NOTE

Quantum Circuit Design of Josephson Traveling Wave Parametric Amplifiers

Understanding the intricate nature of Josephson junctions and the necessity for meticulous control over various parameters to attain optimal performance

KEYSIGHT

Table of Contents

Quantum Circuit Design of Josephson Traveling Wave Parametric Amplifiers 3

Quantum Amplifiers in Superconducting Quantum Systems 4

Introduction to Josephson Junction Traveling Wave Parametric Amplifiers 6

Resonant versus Traveling Wave Parametric Amplifiers 7

Workflow for the Circuit Design of JTWPAs 8

JTWPAs Design with ADS 9

References 16

Quantum Circuit Design of Josephson Traveling Wave Parametric Amplifiers

Quantum amplifiers play a crucial role in the output stages of many quantum systems, where the signals are often extremely weak and challenging to detect and manipulate without amplification [1]. Quantum-limited amplifiers enhance these weak signals without introducing significant noise or distortion [2], which is vital for maintaining the delicate quantum states of these systems and enhancing readout fidelity. Moreover, amplifying quantum signals is crucial for various applications, such as quantum communication, quantum computing, and precision measurements in physics. For instance, in quantum communication, quantum amplifiers enhance the signal strength of single photons, enabling their transmission over long distances in fiber optic networks. In quantum computing, they amplify the weak signals generated by qubits during readout. In precision measurements, quantum amplifiers also increase the sensitivity of detecting weak electromagnetic fields, gravitational waves, and other physical quantities. Quantum amplifiers are indispensable components in many quantum technologies, significantly contributing to our understanding and control of quantum systems.

Parametric amplification is commonly employed in superconducting quantum amplifiers, leveraging the nonlinear properties of Josephson junctions (JJs) to boost weak quantum signals via interaction with a pump signal [2-4]. This nonlinear process generates new photons at frequencies distinct from the input signal. By fine-tuning the system's parameters, these newly created photons can carry information about the input signal, effectively amplifying it. A key advantage of parametric quantum amplification is its ability to achieve high gain with minimal added noise. It is a valuable tool for enhancing weak quantum signals in various applications.

A Josephson Traveling-Wave Parametric Amplifier (JTWPA) is a specialized device used in superconducting quantum systems to achieve broadband, high-gain amplification of weak quantum signals with minimal added noise [4-6]. Unlike resonator-based amplifiers, which operate over narrow frequency bands, the JTWPA utilizes a long chain of Josephson junctions or a transmission line with embedded nonlinear elements to support traveling electromagnetic waves. This structure allows for phase-matching conditions that enable efficient energy transfer from a strong pump signal to the signal of interest, leading to amplification over a broad frequency range. The nonlinear nature of the Josephson junctions facilitates the generation of signal, idler, and pump interactions, resulting in new frequency components that encode information about the input signal. By carefully designing the dispersion properties of the transmission line, it is possible to achieve phase matching across a wide range of frequencies, making JTWPA an essential tool for broadband quantum signal processing [4]. Its ability to provide significant amplification with low noise has made it a key technology in quantum computing and precision measurement applications, where preserving the fragile quantum state of signals is paramount.

In this application note, we introduce quantum amplifiers in superconducting systems, focusing on traveling wave parametric amplifiers (TWPAs). We compare resonant and traveling wave amplifiers and outline the design process for Josephson Traveling Wave Parametric Amplifiers (JTWPAs), which comprise thousands of JJ units. Using nonlinear analysis in ADS, we demonstrate a step-by-step method to study parametric amplification at the circuit level. We also highlight ADS's optimization capabilities for achieving optimal gain over the desired frequency range and present two significant JTWPA designs, uniform and Floquet, based on the critical current profile of the JJs.

Quantum Amplifiers in Superconducting Quantum Systems

In superconducting quantum systems, as shown in Figure 1, quantum operations occur at millikelvin temperatures inside a Dilution Fridge. The RF signal from the Control/Measurement Unit (green rectangle) is attenuated to remove thermal noise before entering the superconducting circuits. After quantum operations, the low-power signal requires significant amplification to avoid thermal noise interference. Quantum amplifiers operate near the **Standard Quantum Limit (SQL)**, the fundamental limit on measurement precision imposed by quantum mechanics, driven by the Heisenberg uncertainty principle. In quantum amplifiers, the SQL dictates that at least half a photon of noise must be added to the signal due to unavoidable quantum fluctuations in the vacuum state [2]. So, for a signal of frequency f, The noise corresponding to the SQL can be expressed as:

$$Noise\ Power = \frac{1}{2}hf$$

Surpassing the SQL requires advanced techniques, such as squeezed states or quantum non-demolition measurements, which enable greater precision in quantum sensing and metrology.

Quantum amplifiers can contain a few to several thousand Josephson junctions. These amplifiers function through the nonlinear mixing of the pump ω_{pump}, the idler ω_{idler}, and the signal ω_{signal}. The pump is a high-power signal that provides energy to the system, amplifying the weaker signal. In this process, an additional frequency component called the idler is generated due to energy conservation and frequency mixing and represents the internal resonance of the quantum amplifier.

Depending on the nature of the nonlinearity, parametric amplification can occur via two main categories of wave mixing:

Three-wave mixing (3WM): In three-wave mixing, two of the three interacting waves (typically the pump and signal) combine to generate the idler, with the frequencies satisfying the relation

$$\omega_{pump} = \omega_{idler} + \omega_{signal}$$

Four-wave mixing (4WM): In four-wave mixing, the interaction involves four waves, typically two pump waves, a signal, and an idler, such that twice the pump tone equals the two output tones

$$2\omega_{pump} = \omega_{idler} + \omega_{signal}$$

Nonlinear mixing processes transfer energy from the pump to the signal and idler, amplifying the signal. Parametric amplifiers use phase and frequency control to achieve high-gain, low-noise amplification, which is essential for quantum technologies, optical communications, and microwave signal processing.

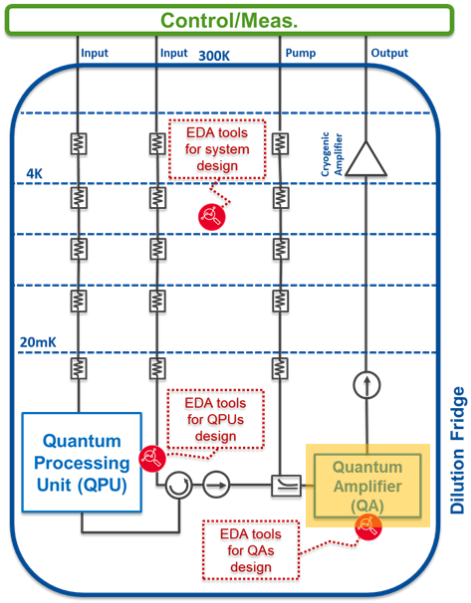

Figure 1. A generic diagram depicting the signal flow in a superconducting qubit experimental setup. The signal from the Quantum Processing Unit (QPU) is amplified by a Quantum Amplifier (QA).

Circuit simulation tools play a critical role in designing and optimizing quantum amplifiers by enabling accurate modeling of nonlinear effects, impedance matching, and signal propagation [3, 6]. These tools allow engineers to simulate the behavior of Josephson junctions, transmission lines, and parametric interactions before fabrication, significantly reducing design cycles and experimental costs. They provide insights into key performance metrics like gain, bandwidth, and noise figures while facilitating the design of phase-matching conditions essential for efficient three-wave and four-wave mixing. By predicting system behavior under realistic conditions, circuit simulators support the development of high-gain, low-noise amplifiers crucial for quantum computing, sensing, and metrology. This application note explores using ADS for designing JTWPA quantum amplifiers with over 2000 Josephson junctions, demonstrating linear and nonlinear analyses to optimize circuit performance.

Introduction to Josephson Junction Traveling Wave Parametric Amplifiers

The Josephson Traveling-Wave Parametric Amplifier (JTWPA) stands out among other types of quantum amplifiers due to its unique properties and operational principles in terms of:

Principle of Operation: The JTWPA leverages the nonlinear properties of Josephson junctions to achieve amplification through mostly four-wave mixing.

Noise Performance: One of the JTWPA's key advantages is its ability to achieve high gain with minimal added noise, making it highly suitable for quantum applications.

Bandwidth: The JTWPA relies on traveling wave structure to offer broad bandwidth, which is beneficial for applications requiring amplifying a wide range of frequencies.

Figure 2 showcases an example of a JTWPA from Ref. [5] with a few GHz gain bandwidth and the stopband appearing at 7.1 GHz. As shown in the lumped element representation, the Josephson junction inductance and shunt capacitance give a well-matched characteristic impedance, and the resonators placed every few JJ units allow for good phase-matching in the four-wave mixing process.

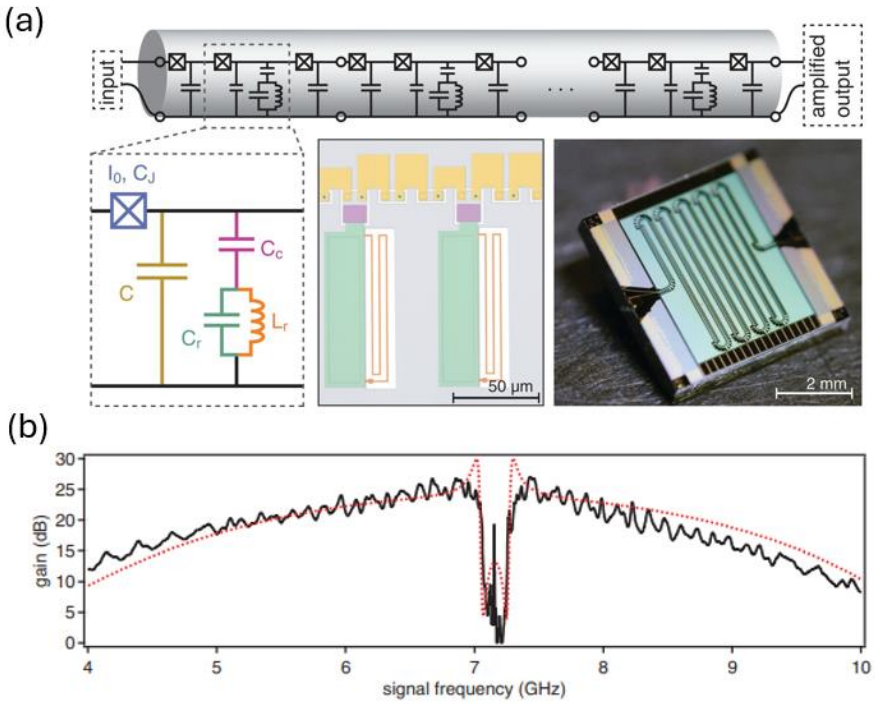

Figure 2. Example of JTWPA from Ref. [5]. (a) Arrangement of Josephson junctions and shunt capacitors for an impedance-matched distributed transmission line. The inset shows the lumped element representations of a line segment, an optical micrograph of the transmission line segment, and the fabricated device. (b) the gain response of the JTWPA with its unique wide range of amplification along with the signature of the dispersive feature at 7.1 GHz.

Resonant versus Traveling Wave Parametric Amplifiers

Quantum amplifiers can be categorized based on their wave propagation into:

Resonant structures: On one hand, a resonant structure relies on resonant circuits, such as LC circuits or resonant cavities, to amplify signals at specific frequencies. These structures are designed to resonate at particular frequencies, providing high gain at those frequencies but limited bandwidth. The resonant nature of these amplifiers means that they are highly selective, amplifying only a narrow range of frequencies around the resonant point. Josephson Parametric Amplifiers (JPAs [2]) and Kinetic Inductance Parametric Amplifier (KIPA [7]) are examples of resonant quantum amplifiers.

Traveling wave structures: In a traveling wave structure, the signal propagates continuously along a transmission line, interacting with a series of nonlinear elements, such as Josephson junctions, to achieve amplification. This structure allows for a broad bandwidth because the signal is not confined to a specific resonant frequency. The continuous propagation of the signal means that it can be amplified over a wide range of frequencies, making traveling wave amplifiers ideal for applications requiring broad frequency coverage. JTWPA [4] and Kinetic Inductance Traveling-Wave Amplifiers (KITWAs [8]) are examples of traveling-wave quantum amplifiers.

Table 1 compares the two structures. In summary, resonant quantum amplifiers use lumped-element circuits with a fixed frequency response, making them suitable for narrowband applications like qubit readout. They have a localized nonlinearity and natural phase matching at resonance, simplifying control. In contrast, traveling-wave amplifiers, such as JTWPAs, provide broadband amplification but are more complex. They require precise dispersion engineering and distributed nonlinearities along long transmission lines. While challenging to design and fabricate due to their scalability and the need for uniform placement of nonlinear elements, they are necessary for applications that demand multi-frequency or broadband signal amplification.

Table 1. Resonant versus traveling wave quantum amplifiers

	Resonant structures	Travelling-wave structures
Design complexity	Simple (lumped-element)	Complex (distributed transmission line)
Frequency range	Narrowband (single mode)	Broadband (multi-GHz range)
Phase matching	Easy (natural resonance)	Difficult (dispersion engineering)
Scalability	Limited to fixed bands	Scalable to broadband
Fabrication	Simple (few components)	Complex (many junctions)
Examples	JPA, KIPA	JTWPA, KITWPA
Application	Single-frequency qubit readout	Broadband quantum signal processing

Workflow for the Circuit Design of JTWPAs

Designing a Josephson Traveling-Wave Parametric Amplifier (JTWPA) involves several key steps to ensure optimal performance, as shown in Figure 3. Here's a detailed workflow:

First, **Build the Circuit** by creating the initial design of the JTWPA, which includes the Josephson junctions and the associated lumped circuit components. In this step, designers may build a unit cell and multiply them to mimic the repetitive LC elements in a transmission line (uniform JTWPA) or import a netlist file containing all circuit elements (Floquet JTWPA). In the Floquet JTWPA, the inductance of each JJ unit differs, reflecting the change of the critical current, which necessitates adjusting the shunt capacitance accordingly to maintain a constant characteristic impedance throughout the transmission line.

Next, **Find the Resonance Feature Frequency** by analyzing the circuit to identify the resonance frequency. This involves performing frequency domain analysis to determine the system's natural resonant frequency.

Then, **Tune the Resonance Feature to the Desired Value** by adjusting the parameters of the circuit, such as the inductance and capacitance, to shift the resonance frequency to the desired value. This step ensures that the amplifier operates at the optimal frequency for the intended application.

After that, set the **Initial Settings for the Pump (Frequency and Power)**. This involves selecting the appropriate pump frequency and power level for the parametric amplification process. The pump signal is crucial for generating the nonlinear effects needed for amplification.

Next, **Maximize the Gain with Pump Power** by gradually increasing the pump power while monitoring the amplifier's gain. The goal is to achieve the highest possible gain without introducing excessive noise or instability.

Finally, **Optimize Signal Power to Avoid Compression** by adjusting the input signal power to ensure the amplifier operates within its linear range. This step is essential to prevent signal distortion and maintain high fidelity in the amplified output.

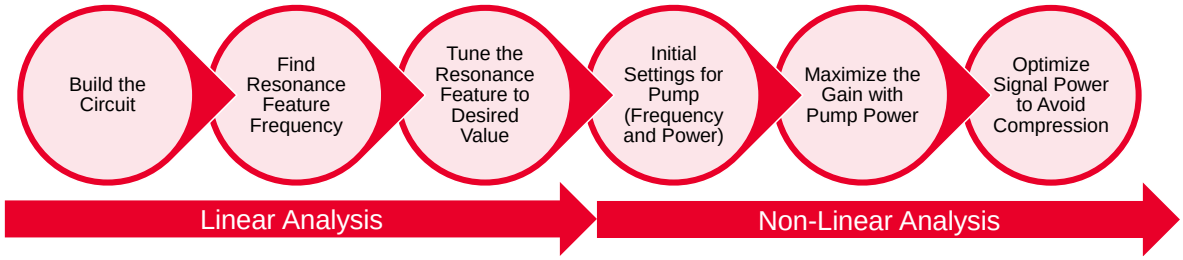

Figure 3. Workflow for designing JTWPA. The whole process can be divided into linear and nonlinear analyses. The former focuses on finding the resonance feature while the latter emphasizes finding a desirable gain over the frequencies of interest and exploring other nonlinear effects.

Following these steps, you can design a JTWPA that provides high gain, low noise, and broad bandwidth, making it suitable for various quantum applications.

JTWPAs Design with ADS

This section demonstrates the analysis of JTWPAs in ADS. We examine both uniform and Floquet (non-uniform) JTWPA structures. In the uniform design, thousands of repetitive LC elements with identical parameters are utilized. Conversely, the Floquet design features each element with distinct parameters. The design process for the JTWPAs here involves a four-wave mixing (4WM) ($2\omega_{pump} = \omega_{idler} + \omega_{signal}$

Due to its nonlinearity, JTWPA creates higher harmonics (3rd, 5th, 7th, etc.). In both cases, we can construct circuits and use the workflow to optimize the JTWPA's gain performance and bandwidth.

Uniform JTWPA

Figure 4(a) illustrates the uniform JTWPA circuit, comprising 2048 Josephson junctions. Each junction has a critical current of 3.4 μA, a shunt capacitance of 45 fF, and a shunt resistance of 2.91 MΩ. These junctions are organized into 512 repetitions of the four-junction unit cell depicted in Figure 4(b), forming the complete amplifier structure as shown in the S-Parameters linear setup in Figure 4(c). To ensure good phase-matching in the FWM process, LC resonators are periodically placed in the circuit. An LC resonator with L = 170 pH and C = 2815 fF is placed after every four JJ units in the uniform JTWPA circuit.

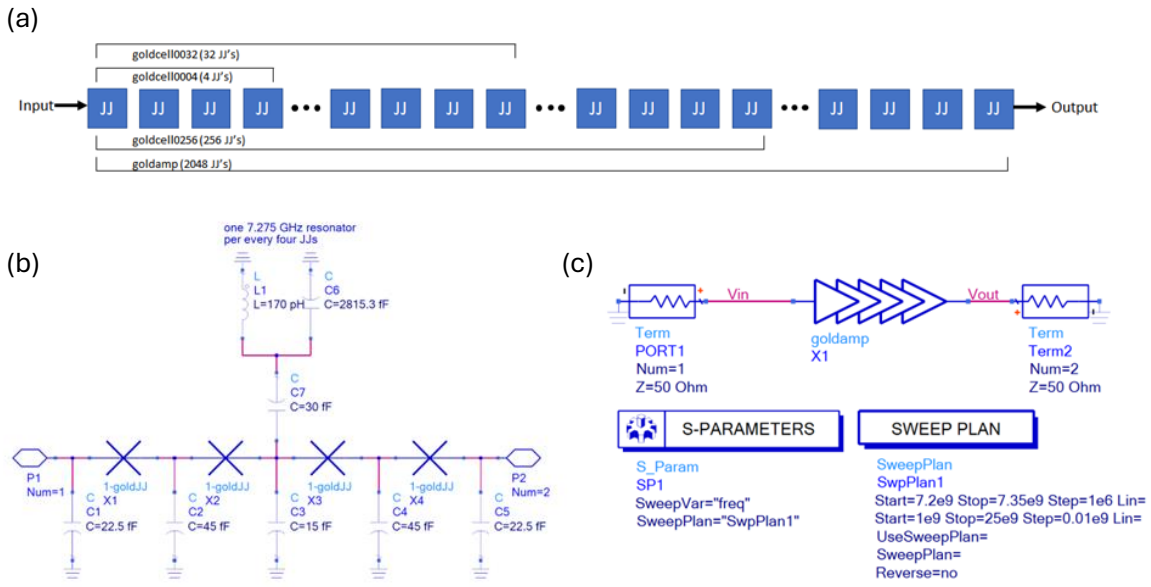

Figure 4. Uniform JTWPA. (a) Arrangement of JJs. (b) The four-junction unit cell. (c) Full amplifier setup for the linear S-Parameters analysis.

We will first explore the linear response of the amplifier to find the frequency of the JTWPA's resonance feature. Then, we will search for the parameters to get optimal parametric amplification at 6 GHz. The linear analysis is a frequency sweep of the circuit without any pump current to ensure the circuit behaves well over the frequency of interest. Since resonators are placed periodically in the transmission line to compensate for the phase differences between the different tones, they give rise to a stopband (dispersive feature) in the circuit's frequency response, as shown in Figure 5(a). It is essential at this stage to tune the resonators' parameters to ensure the circuit stopband does not overlap with the frequency range of interest. Simulation is carried out over a frequency span from 1 GHz to 25 GHz in 10 MHz frequency steps. Another sweep is carried out from 7.2 GHz to 7.35 GHz in 1 MHz steps for a finer frequency analysis near the resonant frequency, as shown in Figure 5(a) and (b), respectively. The plots below demonstrate a distinct resonance feature in the frequency response, characterized by near-zero transmission and complete reflection at 7.24 GHz.

We then add the pump current to the circuit and utilize Harmonic Balance to investigate the nonlinear response of the JTWPA, as shown in Figure 6(a). Here, we analyze the nonlinear frequency response with fixed signal and pump frequencies. The pump signal is at 7.12 GHz, set slightly lower than the resonant feature, and its current at 3.4 uA is initially set close to the critical current of JJ's. Meanwhile, the signal tone is fixed at 6 GHz with a power of -150 dBm. The Harmonic Balance solver settings include two fundamental frequencies: Fpump with an order of 15 and Fsig with an order of 3.

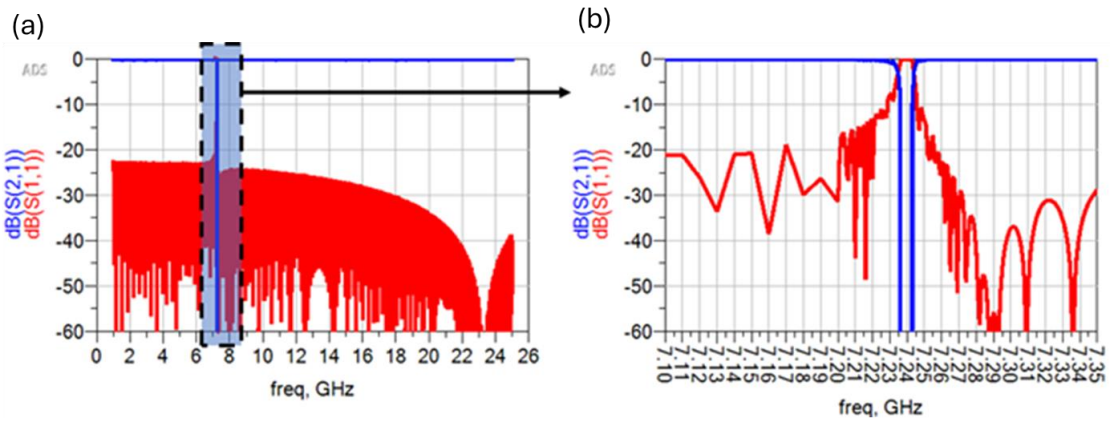

Figure 5. Linear S-Parameters analysis (in the absence of the 4WM process) to adjust the frequency of the dispersive feature (minimum transmission) away from the frequency range of interest. (a) Broadband response. (b) Zoomed in response to the dispersive feature at 7.24GHz.

The resulting output signal spectrum is shown in Figure 6(b). The fundamental output pump signal (f_{pump}) is at 7.12 GHz, along with the third, fifth, and seventh harmonics at 21.36 GHz, 35.6 GHz, and 49.84 GHz, respectively. As expected, the even harmonics are not naturally excited and do not appear in this amplifier because of the intrinsic properties of the Josephson junctions, which exhibit a current-phase relationship, $I = I_c\, sin(\phi)$, which is inherently nonlinear and primarily contributes to odd-order harmonics $sin(\phi) = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} + \cdots$

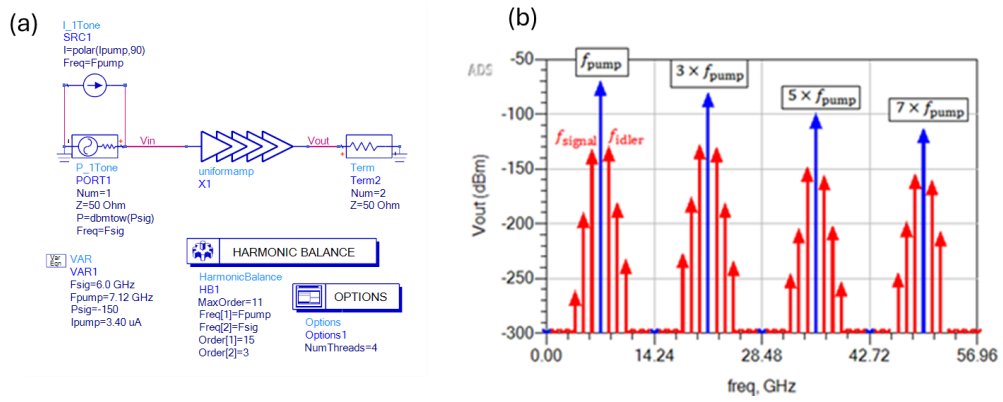

Figure 6. Nonlinear response with ADS Harmonic Balance. (a) Circuit setup. (b) Mixing products of the harmonics.

Next, we perform a frequency sweep of the input signal, shown in Figure 7(a), with a fixed pump signal frequency (7.12 GHz) and current (3.4 uA). Using a small signal power of -150 dBm, we sweep from 100 MHz to 14 GHz. The Harmonic Balance settings remain unchanged. The initial gain response versus input signal frequency is displayed in Figure 7(b), showing a broadband range with around 15 dB gain at the initial pump settings. As expected, a dip in gain appears around the JTWPA's resonance feature at 7.12 GHz.

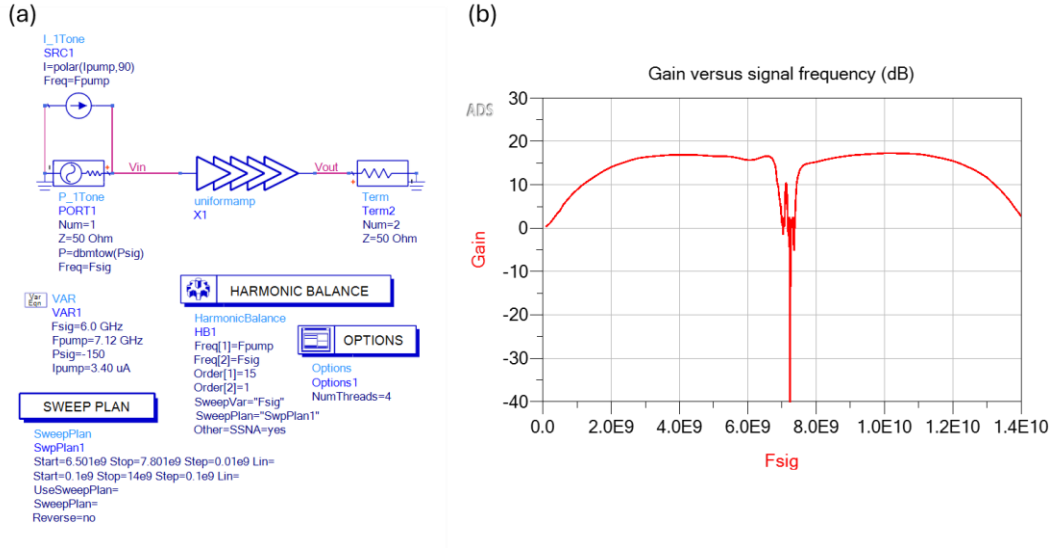

Figure 7. Initial gain response of the uniform JTWPA with Harmonic Balance. (a) Circuit setup. (b) Optimized gain response versus frequency.

Gain optimization can be performed in two steps. First, parameter optimization determines the pump current that achieves the target gain at the desired signal frequency, as illustrated in Figure 8(a). This pump current is then used in the circuit setup of Figure 8(b) to generate the final optimized gain curve over the frequency span, as shown in Figure 8(c). To perform optimization, the circuit requires a nominal optimization block (OPTIM), an optimization goal (Goal), and a current sweep in variables (VAR).

The optimization goal is to achieve a 25 dB gain at 6 GHz. This corresponds to dBm(mix(Vout,{0,1}))+150. Since the input signal has -150 dBm power, subtracting this from the output power determines the amplifier's gain. By setting the min = max = 25 dB, we look for a condition where the gain becomes 25 dB. Meanwhile, in the variable window (VAR), we allow the pump current to vary in the range of 3 µA to 3.7 µA during optimization. Lastly, we configure the optimizer (Optim) with OptimType as Gradient with MaxIters of 25. Following a successful optimization process, the target signal gain of 25 dB at 6 GHz was attained with a pump current (Ipump) of 3.681 µA, as illustrated in Figure 8(b) and (c).

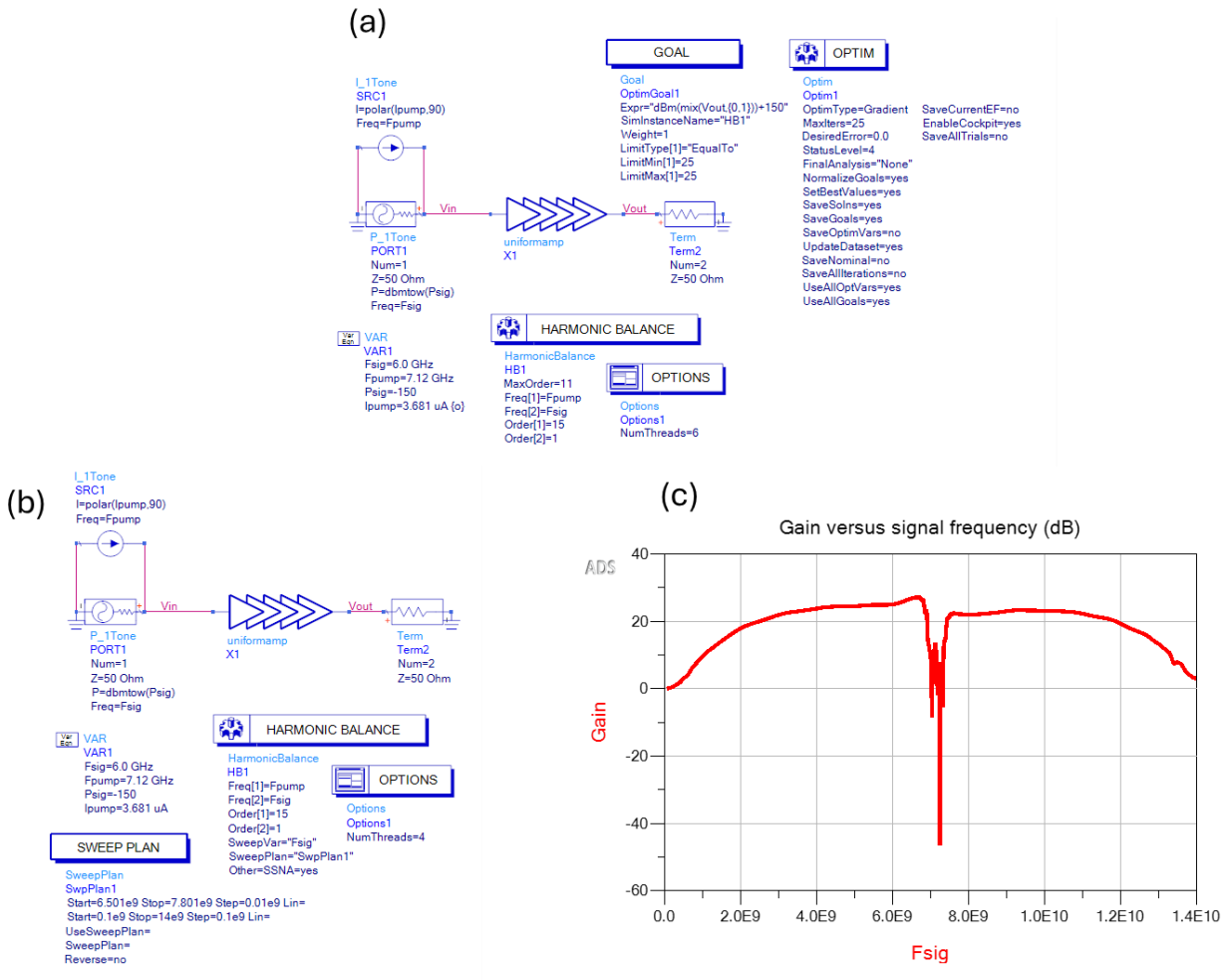

Figure 8. Optimized gain response of the uniform JTWPA with Harmonic Balance. (a) Optimization. (b) Circuit setup with optimized pump parameters. (c) Optimized gain response versus frequency.

As a last step in the design process of the JTWPA, we investigate the amplifier's saturation by sweeping the input power, as shown in Figure 9(a). We aim to determine when gain compression begins at 6.0 GHz for the input signal, with the pump signal set at 7.12 GHz and a current of 3.681 µA. For that, we run the Harmonic balance analysis while sweeping the signal power from -150 dBm to -114 dBm in increments of 2 dB. The power sweep simulation results are shown for the gain in Figure 9(b) and the output signal harmonics (fundamental in red and 3rd harmonic in magenta) in Figure 9(c). As the input power level increases, the gain starts noticeably decreasing, and the 3rd harmonic power content relative to the fundamental one increases. At -114 dB, the gain is compressed by about 0.5 dB. Simulating slightly higher signal power is possible, but since it causes the amplifier to become highly nonlinear, the simulation tends to take longer to converge.

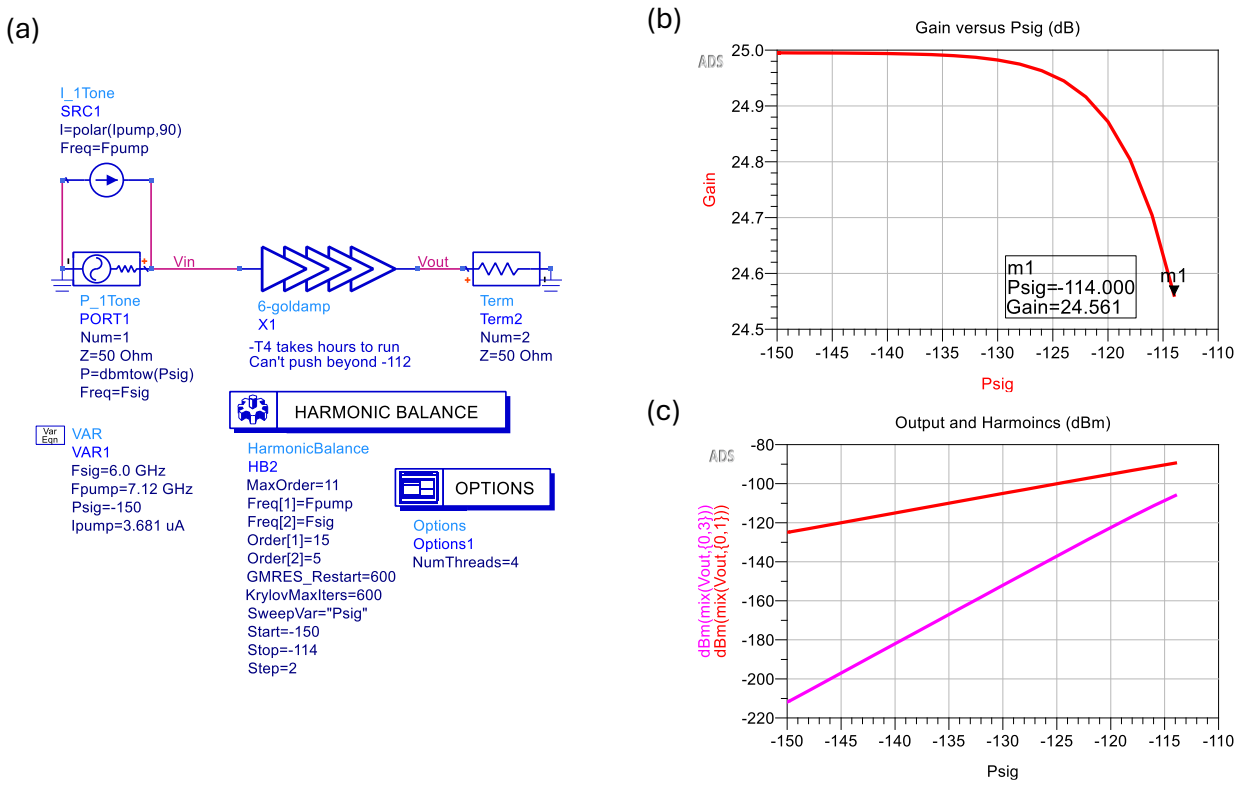

Figure 9. Gain compression analysis of the uniform JTPWA. (a) Circuit setup. (b) Gain versus signal power, demonstrating the compression of the amplifier starting from a signal power of -130 dBm. (c) The first and third harmonics, showing the increase of the third harmonic upon compressing the mplifier.

Floquet JTWPA

Unlike the uniform JTWPA, the Floquet design shown in Figure 10(a) features a profile for the critical current of the JJ along the TWPA geometry, as illustrated in Figure 10(b). The critical current of the first junction is 10.6 uA, decreasing parabolically to 1.75 uA at the center, then increasing back to 10.6 uA at the other end of the JTWPA. To achieve impedance matching, the shunt capacitance must vary by position to maintain a characteristic impedance of 50 Ohms. An LC resonator with L = 1533 fH and C = 247 pF is placed after every eight JJ units, which leads to a stopband (dispersive feature) at around 8.2 GHz. The Floquet JTWPA design has a total of 1999 JJ units compared to 2048 JJ units in the case of the uniform one.

The entire amplifier circuit can be outlined in a netlist file and subjected to a design process similar to that used in the uniform case. This process involves starting with linear analysis using S-parameters to confirm the location of the dispersive feature, followed by nonlinear analysis to identify the mixing products and initial gain. The final steps involve optimizing the gain and examining gain compression, as illustrated in the workflow shown in Figure 3. This section presents the results for the optimized gain response. It compares the responses of the Floquet and uniform designs, as illustrated in Figure 10(c).

Notably, the Floquet design's gain response exhibits a significantly smoother profile, benefiting from the continuous variations in the critical current of the junctions. Conversely, the uniform JTWPA demonstrates a wider operating frequency range, maintaining a measurable signal gain even at 2 GHz.

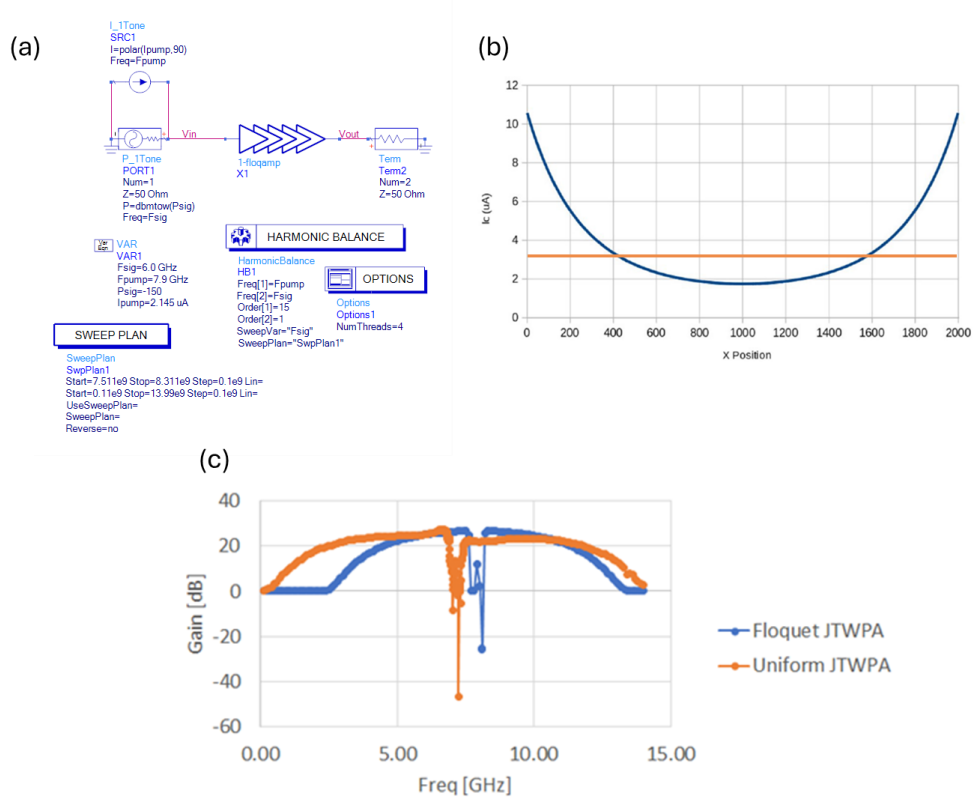

Figure 10. Floquet JTWPA. (a) Circuit setup for gain analysis. Comparison of the Floquet and uniform JTWPA schemes in terms of (b) critical current profile for the JJs, and (c) gain responses.

Table 2 summarizes the design parameters for both the Floquet and uniform designs to achieve a parametric gain of 25 dB at 6 GHz. The Floquet JTWPA has approximately fifty fewer Josephson junctions and fewer phase-matching resonators, and it requires a lower pump power of 2.2 µA compared to the uniform JTWPA, which needs a pump current of 3.7 µA. The pump frequency for the Floquet design is 7.9 GHz, while it is 7.12 GHz for the uniform design.

Table 2. Comparison of the Floquet and uniform JTWPA designs.

	Floquet	Uniform
Number of JJ's	1999	2048
Pump freq	7.9 GHz	7.12 GHz
Pump current	2.2 µA	3.7 µA
Pump power	-75.17 dBm	-70.68 dBm
Critical current	1.75 - 10.60 µA	3.40 µA

Conclusion

Josephson Traveling Wave Parametric Amplifiers (JTWPAs) represent a notable advancement in quantum electronics. By utilizing the distinctive properties of Josephson junctions and traveling wave structures, JTWPAs provide high gain, low noise, and wide bandwidth, making them suitable for applications in quantum computing and precise microwave measurements. The design complexity associated with JTWPAs arises from the intricate nature of Josephson junctions and the necessity for meticulous control over various parameters to attain optimal performance. Electronic Design Automation (EDA) tools are essential to address this complexity. EDA tools provide advanced simulation capabilities that enable designers to conduct linear and non-linear analyses and optimize JTWPA performance to meet requirements.

References

1. Krantz et al. "A quantum engineer's guide to superconducting qubits." Applied Physics Reviews 6.2 (2019): 021318.
2. Aumentado. "Superconducting parametric amplifiers: The state of the art in Josephson parametric amplifiers." *IEEE Microwave Magazine* 21.8 (2020): 45-59.
3. Daryoush et al. "Modeling and Harmonic Balance Analysis of Superconducting Parametric Amplifiers for Qubit Readout: A Tutorial." *IEEE Microwave Magazine* 25.11 (2024): 54-73.
4. O'Brien et al. "Resonant phase matching of Josephson junction traveling wave parametric amplifiers." *Physical review letters* 113.15 (2014): 157001.
5. Macklin et al. "A near-quantum-limited Josephson traveling-wave parametric amplifier." *Science* 350.6258 (2015): 307-310.
6. Kaidong et al. "X-parameter based design and simulation of Josephson traveling-wave parametric amplifiers for quantum computing applications." 2022 IEEE International Conference on Quantum Computing and Engineering (QCE). IEEE, 2022.
7. Mingrui et al. "Magnetic field-resilient quantum-limited parametric amplifier." *PRX Quantum* 4.1 (2023): 010322.

Sweetnam et al. "Simulating the behavior of traveling wave superconducting parametric amplifiers using a commercial circuit simulator." *Superconductor Science and Technology* 35.9 (2022): 095011.

Keysight enables innovators to push the boundaries of engineering by quickly solving design, emulation, and test challenges to create the best product experiences. Start your innovation journey at www.keysight.com.

KEYSIGHT

This information is subject to change without notice. © Keysight Technologies, 2025, Published in USA, January 23, 2025, 3125-1012.EN